

SIMPLE FORMULAS AND GRAPHS FOR DESIGN OF VENTED LOUDSPEAKER SYSTEMS

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*4th order &
6th order vented*

*to design Bass Rfx
Spkrs*

SIMPLE FORMULAS AND GRAPHS FOR DESIGN OF VENTED LOUDSPEAKER SYSTEMS

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This paper gives formulas for designing both fourth-order and sixth-order (equalized) systems for a given woofer. The formulas specify the frequency to which the box must be tuned, based on the woofer parameters. The designer has a choice of box volumes and cutoff frequencies (which are related). Graphs of the frequency response (versus box volume and woofer parameters) are also given. The graphs are scaled in units of f_s/Q_t and $V_{as}Q_t^2$, which makes them applicable to a very wide range of woofers and box sizes.

INTRODUCTION

The problem of finding the frequency response curve for a vented (bass reflex) loudspeaker system was first solved, for readers in this country at least, by the reprinting of Neville Thiele's classic paper "Loudspeakers in Vented Boxes" in the Journal of the Audio Engineering Society in 1971 (Ref. 1). His equation (12) gives the system frequency response function based on the physical parameters of the woofer and enclosure used.

Thiele simplified the problem by assuming that the enclosure Q (Q_b), that is, the Q associated with the resonance of the air mass in the vent with the compliance of the volume of air in the box, is infinite. Small gives the system response function taking into account the enclosure Q (Ref. 2, equation 13).

One might think that the "mother lode" of information provided by the system response function would have been pretty well mined out by now by the many investigators who have worked on this problem. Not so.

While measuring the properties of woofers that our company produces, I observed that while there was considerable unit-to-unit variation in the parameters of woofers of the same model, these variations had little effect on the acoustic performance of the systems in which they were used. This suggested to me that the performance of loudspeaker systems is actually determined by one or more relatively invariant properties of the woofers, while the commonly measured parameters defined above

are unduly influenced by some varying factor that really has little effect on performance.

MATHEMATICAL ANALYSIS

The system frequency response function relates the relative sound pressure level output of a loudspeaker system, $E(s)$, to the complex frequency variable, s . "Relative sound pressure level" means that the sound pressure level at high frequencies is assumed to approach unity, so that the function actually expresses the low-frequency output as compared to the higher-frequency output of the same driver-in-box. This corresponds to what is commonly called the "woofer section frequency response" of a multiple-driver speaker system. (This paper is solely concerned with the low-end response design problems.)

The response function is

$$E(s) = \frac{s^4}{\left[s^4 + \left(\frac{f_s}{Q_t} + \frac{f_b}{Q_b} \right) s^3 + \left(\frac{V_{as}f_s^2}{V_b} + f_s^2 + \frac{f_b f_s}{Q_b Q_t} + f_b^2 \right) s^2 + \left(\frac{f_b f_s^2}{Q_b} + \frac{f_b^2 f_s}{Q_t} \right) s + f_b^2 f_s^2 \right]} \quad (1)$$

The frequency response is controlled by the six parameters in the function, three for the woofer and three for the box:

f_s	natural resonant frequency of the woofer (measured on a flat baffle)
Q_t	total Q of the woofer in the system at f_s , including all electrical and mechanical resistances, which I assume throughout this paper to equal Q_{ts} , the total Q of the woofer itself
V_{as}	volume of air having same acoustic compliance as driver suspension
f_b	frequency of enclosure resonance (resonance of air mass in vent with compliance of air volume in box)
Q_b	total box Q at f_b due to all enclosure losses; assumed throughout this paper equal to 7
V_b	net volume of air inside enclosure

The driver resonant frequency, f_s , is usually defined at the free-air (unen-

closed) resonant frequency, but I prefer to use the resonant frequency as measured on a flat baffle because this method includes the effects of an air mass load more like the air load the driver experiences in an enclosure. Therefore frequency response calculations based on f_s defined and measured this way should be more accurate.

Under certain circumstances Q_t will differ from Q_{ts} . If damping material is packed close around the back of the woofer in the enclosure, the added resistance to air flow will change the woofer's effective Q . Also making the source impedance driving the woofer other than zero--for example by interposing resistance between the amplifier and woofer, or by designing as special negative-output-impedance amplifier--will change the value of Q (Ref. 5). However, ordinary high-damping-factor transistor amplifiers have near-zero output impedance, and so long as you have a low-resistance connection from the amp to the woofer the Q will be undisturbed; Q_t will equal Q_{ts} . (The marking "4-8 ohms" that you see on the back of amps means "attach a 4-8 ohm load here," not that the amplifier's output impedance is itself equal to 4-8 ohms.) I have assumed throughout this paper that Q_t and Q_{ts} are interchangeable. If you ever do run into a case where they differ, Q_t is the proper number to use in the formulas and graphs presented here.

The box Q , Q_b , is assumed here to be made up solely of leakage losses; losses of any other kind would change the form of eq. (1) slightly. I also assume throughout that Q_b is equal to 7. These two assumptions give results that are in reasonable conformity with practical speaker enclosure construction methods (Ref. 2).

There are other important driver parameters, such as the maximum power handling ability and the efficiency, that are not considered at all in this paper. Only factors that affect frequency response are considered. I would like to point out, however, that once a driver is selected, the enclosure design has no effect on system efficiency. The well-known dictum that "vented systems are more efficient than sealed systems" really means that a woofer optimized for use in a vented box of a certain size is more efficient than a woofer optimized for use in the same box sealed. This is because vented systems can use woofers of lower Q and still get good response shapes, and lower Q is caused by higher motor strength (larger magnet). Hence higher efficiency.

Keele observed that driver compliance V_{as} has relatively little effect on system frequency response (Ref. 3). I also know that of the woofer physical parameters--compliance, cone mass, magnet strength, and so on--compliance is the one that varies the most in production. An examination of driver resonant frequency and Q reveals that these two factors contain the compliance implicitly. The resonant frequency is sensitive to the compliance, of course, because it is determined by the cone mass and compliance. The Q is in turn influenced by the compliance because it is

measured at the resonant frequency.

The undesired sensitivity to compliance may be "factored out" of the Q by choosing to describe the woofer's Q by

$$f_s/Q_t$$

rather than simply by Q_t . This factor may be analyzed by substituting the mechanical parameters for the acoustic parameters:

$$\frac{f_s}{Q_t} = \frac{2\pi\omega_s}{\omega_s M_{ms}/R_{ls}} = \frac{2\pi R_{ms}}{M_{ms}} \quad (2)$$

ω_s is the resonant frequency in radians/second, R_{ms} is the total driver cone resistance in mechanical units, and M_{ms} is the cone mass in mechanical units. (For a complete description of the mechanical parameters and their effects, see Ref. 1 or 2.) You can see that the frequency, and therefore the dependence on compliance, cancels out.

Similarly we can use as a descriptive woofer parameter in place of V_{as} the factor:

$$V_{as} f_s^2$$

That this factor does not really have any dependence on compliance may again be demonstrated by expressing it in other parameters:

$$V_{as} f_s^2 = V_{as} \left(\frac{1}{4\pi^2 V_{as} M_{as}} \right) = \frac{1}{4\pi^2 M_{as}} \quad (3)$$

where M_{as} is the acoustic mass of the woofer. Thus the compliance cancels out and the factor $V_{as} f_s^2$ is really an expression of the acoustic mass of the woofer (or of the reciprocal of the mass, to be more exact).

Thus we may, if we wish, replace the three usual woofer parameters

$$f_s, Q_t, \text{ and } V_{as} \quad (4)$$

with the "more fundamental" set

$$f_s/Q_t, V_{as} f_s^2, \text{ and } V_{as} \quad (5)$$

The parameter V_{as} has no dependence on the other two so it can stay the same.

The "fundamentalness" of f_s/Q_t and $V_{as}f_s^2$ may be confirmed by re-examining the system response function, eq. (1). You can see that Q_t appears in three places, each time in the factor f_s/Q_t . The compliance V_{as} appears only once, and then in the factor $V_{as}f_s^2$.

This might suggest that loudspeaker parameters be specified in terms of the new set of factors (5) rather than the usual set (4). As a practical matter this is not necessary. Imagine that you have in hand a woofer manufactured with the correct cone mass and motor strength but the compliance is off by a certain amount. The true values of f_s , Q_t , and V_{as} for your woofer will all differ from the values specified by the manufacturer. However, the values of the factors f_s/Q_t and $V_{as}f_s^2$ as calculated from the specs will be correct for your woofer because it has the right cone mass and motor strength. Errors in f_s and Q_t caused by incorrect V_{as} will cancel when the parameters are combined in f_s/Q_t and $V_{as}f_s^2$. Your own calculation of these two factors will give you the same numbers as the manufacturer would have given you anyway.

To the woofer designer, however, for whom the motor strength, cone mass, and compliance are variables that he controls independently, the new set of woofer parameters (5) may be very useful.

* BOLD CONJECTURE

After seeing how the new parameter set fit nicely into eq. (1), I turned to Thiele's well-known table of loudspeaker alignments (Table I, Ref. 1; with minor corrections in Ref. 3) to see if restating the column headings in terms of the new factors might lead to any simplifications. To my surprise I found on the right-hand side of the table a column of "approximately constant quantities" for the first nine Alignments. Thiele had observed that

$$\frac{V_{as}f_s^2}{V_b f_3^2} \approx 1.41 \quad (6)$$

$$\frac{Q_t}{f_s} f_b = .39 \quad (7)$$

(My symbols here are slightly different than those in the original table.) f_3 is the -3 dB bass cutoff frequency.

Thus Thiele has uncovered two simple approximations relating f_3 and f_b , the frequency to which the box is tuned, to the three driver parameters and the box volume. Note that with Q_b assumed equal to 7, all six of the frequency-response-determining parameters in the system response function are accounted for. If we make the bold conjecture that this is all the information needed to design a vented speaker system, we can recast eq. (6) and (7) into two design formulas:

$$f_3 = \sqrt{\frac{V_{as}f_s^2}{1.41 V_b}} = .84 \sqrt{\frac{V_{as}f_s^2}{V_b}} \quad (8)$$

$$f_b = .39 \frac{f_s}{Q_t} \quad (9)$$

The design procedure is very simple:

- 1) Pick a convenient size box, V_b .
- 2) Calculate the low-end cutoff (-3 dB) frequency it will give you, f_3 , from eq. (8). If dissatisfied, go back to 1). This time pick a bigger box.
- 3) Tune the box resonant frequency, f_b , to the value specified by eq. (9).

The boldness of this conjecture is that it suggests that you can design a variety of vented systems, using different box volumes and getting different cutoff frequencies, for a given woofer. This is exactly what the loudspeaker system designer—who often has only a limited choice of drivers to pick from—would like to do. Most other design methods give only one allowable box volume, box frequency, and cutoff frequency for a given driver (unless you are willing to take extreme measures such as twiddling with Q_t by adjusting the source impedance).

FOURTH-ORDER VENTED SPEAKER SYSTEM RESPONSE GRAPHS

Formulas (8) and (9) can be tested by substituting the parameter values they prescribe back into the system response equation (1). We can use the response equation to see what sort of frequency response results from systems designed according to the formulas. The actual response calculations are very tedious and repetitive, but a programmable calculator handles them nicely.

The formula for f_b eliminates it as an independent variable, since it is now expressed in terms of other quantities. Assuming Q_b constant at 7 eliminates

-5- * I computed this myself from p. 3 speaker "using Thiele-Small Parameters to Design Loudspeaker Systems"

it as a variable. The number of variables may be further reduced by normalizing, that is, expressing some variables in terms of others.

A normalization scheme that works well is to express frequency in units of f_s/Q_t , that is, to use as the frequency variable

$$\frac{f}{f_s/Q_t}$$

and to express volume in units of $V_{as}Q_t^2$, that is, to use as the volume parameter

$$\frac{V_b}{V_{as}Q_t^2}$$

The details of how and why this is done are explained in the Appendix.

The reduction in the number of variables simplifies the problem so that speaker systems designed for many combinations of f_s , Q_t , V_{as} , and V_b can have their frequency response characteristics expressed in a reasonable number of graphs. These graphs appear in Figures 1 through 7.

Each Figure is for one value of Q_t . Each Figure has several curves for different values of box volume V_b . The curves are labeled for volumes ranging from $2.0 V_{as}Q_t^2$ to $16.0 V_{as}Q_t^2$. The frequency scales are calibrated from $.2 f_s/Q_t$ to $2.0 f_s/Q_t$.

The unusual units will probably make it difficult for most people to conceptualize what the graphs represent. Imagine a "benchmark woofer" with

$$f_s/Q_t = 100 \text{ Hz} \quad \text{and} \quad V_{as}Q_t^2 = 1 \text{ liter or } 1 \text{ ft}^3$$

(A woofer with $f_s = 15.9 \text{ Hz}$, $Q_{TS} = .159$, and $V_{AS} = 39.6 \text{ liters or } 39.6 \text{ ft}^3$ would fill the bill.)

For such a woofer you can read the frequency scale directly in hertz (ignoring the decimal points) from 20 to 200 hertz; and you can read the curves as though labeled simply in liters or ft^3 .

To apply the curves to any other woofer:

- 1) Pick the Figure for Q_t value closest to that of the woofer you are considering.
- 2) Calculate the factor f_s/Q_t for the woofer and multiply the frequency scale numbers by that factor to get the frequency scale in hertz.
- 3) Calculate the factor $V_{as}Q_t^2$ for the woofer and multiply the numbers that label the curves by that factor to get the curve labels in liters or ft^3 .

Examination of these curves reveals some interesting facts. First of all, it is evident that for a given woofer you have a range of useable box volumes and bass cutoff (-3 dB) frequencies, rather than a single "correct" box volume. The cost of this flexibility is some ripple in the response curve. Even allowing only $\pm 1 \text{ dB}$ of ripple, however, the allowable box volumes span a considerable range. For example, a woofer with a Q of .32 can be used in boxes with volumes from 2.0 to 11.3 $V_{as}Q_t^2$ to give cutoff frequencies from .35 to .63 f_s/Q_t . This is a span of almost six to one in box volume, and two to one in cutoff frequency.

Also note that there is little difference in the curves from Figure to Figure; the curves for a woofer Q of .159 (Figure 1) are very similar to the curves for a woofer Q of .20 (Figure 2). This is because the normalization technique "factors out" the unwarranted sensitivity to compliance that injects large variations into families of response curves plotted using other systems of units.

This type of presentation also makes it possible for you to interpolate between the Figures for intermediate values of Q_t if you wish to do so.

The cutoff frequencies for various combinations of woofer Q and box volume are summarized in Table 1. The figures given in the body of the Table are limited to combinations that give $\pm 1 \text{ dB}$ of ripple or less.

Remember that the curves in the Figures and the entries in Table 1 are true only for designs that conform to eq. (9). The fact that eq. (9) yields many designs with reasonable response shapes suggests that it is a worthwhile design tool.

The value of eq. (8) may be assessed by comparing the cutoff frequencies it predicts (which are given in Table 2) against the exact values for the cutoff frequencies calculated using the system response equation (which exact values are given in Table 1). Comparison shows that eq. (8)'s accuracy is rather poor.