
Subject: More Resonances!

Posted by [Martin](#) on Mon, 11 Oct 2004 20:24:18 GMT

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Here is something I wrote for the bass list and Madisound board a few years ago. It is consistent with the response above but adds a few different thoughts. "Please find below a condensed and modified response that I posted on the bass list to a similar question. I have also added some at the bottom to try and address the transient response issue of your question. A similar topic came up on the basslist a few weeks ago. I posted the following responses. When the driver motion hits a minimum, this is the mid point between the two resonances and not a resonance in itself. Hopefully the following will explain, if not feel free to ask some questions, argue a point or two, or just discuss my attempt at a simple explanation of the dynamics and vibration theory. If we restrict the discussion to frequencies between say 1 and 200 Hz, then the impedance can be thought of as a direct reflection of the driver's velocity. The higher the impedance the faster the driver is moving. Let us say that I am trying to measure the impedance of the driver in a bass reflex enclosure, I have decided to probe the device under test with a constant current source. I am not assuming a constant voltage source like an amp. It should not matter because the impedance is a ratio of voltage over current so both test methods will yield the same result. If I start by using a simplified set of equations (ignoring phasor notation) : $f = BL \times i$ (f is mechanical force, i is input current) $e = BL \times u$ (e is back emf, u is velocity) $Z = R_e + e / i = R_e + BL \times BL \times u / f$ (BL is the magnetic flux and the length of wire in the gap product, R_e is the DC resistance) So when you look at the impedance plot, there are the two resonant frequencies of the driver and enclosure combination as seen by the two peaks. If we assume that the amp is applying constant current, then f will also be constant. Therefore, e is a function of u which is determined by the mechanical portion of the driver and enclosure as the speaker is being acted on by a constant applied force f. Hopefully that is not too confusing. If you buy into my simplification of the electrical portion of the speaker, then the solution of the mechanical part for the driver velocity is required to determine the shape of the impedance curve. Suppose that the driver has a resonant frequency f_s as described by the T/S parameters. The resonant frequency of the driver f_s is determined by the stiffness of the suspension (spider and surround) and the moving mass (primarily the cone, voice coil, and former). Also assume that the box has a resonant frequency f_b as determined by the air volume in the box acting as a spring and the air in the port acting as a moving mass. For a standard bass reflex design f_s and f_b are designed to be approximately equal. I am going to ignore damping since it essentially only sets the magnitude of the peaks. The biggest source of damping is in the driver, the enclosure typically is a high Q system so let us assume minimal damping is present in the box. From vibration theory, when you join two resonant systems (mounting the driver in the box) the resulting system natural frequencies will be shifted to bracket the set of individual system resonant frequencies. If the first impedance peak has a frequency of f_1 and the second f_2 , then : $f_1 < f_s \sim f_b < f_2$ I first encountered this observation when reading Lord Rayleigh's Theory of Sound many years ago. The relative motions of the two masses (driver cone and air in the port) can also be described using vibration theory. The two natural frequencies f_1 and f_2 have specific motions (mode shapes) associated with them. For the lower frequency f_1 , the driver and the air in the port move in the same direction and are in phase. The driver moves into the cabinet and the air in the port moves out of the cabinet so as not to overly compress the air in the cabinet, this is what I mean by the same direction. This mode causes the sound to cancel and the 24 dB/octave low end roll-off of a bass reflex design (or TL design). One can think of the air in the port adding an additional mass to the driver's moving mass causing the driver resonant

frequency f_s to drop to f_1 . For the upper frequency f_2 , the driver and the air in the port move in opposite directions and are out of phase significantly compressing the air in the cabinet, this is what I mean by opposite direction. The sound from the driver and the port combine. Remember that I am still at frequencies below 200 Hz, as you move up in frequency the contribution from the port will drop significantly and the driver will produce almost all of the sound. Before going any further, I would like to expand on the mode shape and summation of mode shape representation for the bass reflex speaker. Suppose I have a single degree of freedom mechanical system (like a driver in an infinite baffle) which has a mass denoted by m , some damping denoted by c , and a spring denoted by k . The resonant frequency of this system is $f = 1/(2 \times \pi) \times (k/m)^{1/2}$ in hertz which I hope is no surprise. If I apply a sinusoidal force to the mass m and vary the frequency I can plot a response curve that looks like the impedance curve for a driver in an infinite baffle or closed box. There are three distinct regions of this curve. 1) Below resonance the motion of the mass will be controlled by the stiffness k and will be in phase with the force. 2) At resonance, the damping dominates and the motion will be proportional to the velocity. The force and the displacement will be 90 degrees out of phase. 3) Above resonance, the force is working to accelerate the mass and so the motion will be proportional to the mass. The force and the displacement are 180 degrees out of phase. The force will have a positive sign while the displacement will have a negative sign, the phase has reversed. Please remember these regions of the driver impedance curve and the phase relations between the displacement and the force. Now, returning to the driver in a bass reflex enclosure. As I said before, the two masses are the driver moving mass and the air in the port. The two springs are the driver's suspension and the trapped volume of air in the cabinet between the back of the driver and the entrance to the port. Assuming a simple special case (I am making these numbers up to aid in the visualization so be tolerant please) that by some magic I know the resonant frequencies and mode shapes as shown below: First mode at f_1 has a mode shape $[1.0, 1.2]$ Second mode at f_2 has a mode shape $[1.0, -0.8]$ The numbers in brackets are normalized mode shapes as shown below: $[S_d \times x_d, S_p \times x_p]$ where S_d = driver area x_d = driver displacement positive moving out of the cabinet S_p = port area x_p = port air mass displacement positive moving into the cabinet The numbers in brackets describe the motions of the two masses at the particular frequencies f_1 and f_2 . They are not absolute but are normalized so that the driver has a unit motion. I have selected these values to help illustrate the physics. They are highly idealized so please do not try and draw any absolute conclusions but use them to develop a feel for what is going on. Remember that the impedance curve for the bass reflex design (and the TL design) has two peaks that sort of look like a pair of driver in an infinite baffle impedance curves. Also from mechanical vibration theory, the displacement of any mechanical system can be represented by the linear summation of the mode shapes of that system. Here are some possible combinations that could be used to explain the two peaks in the impedance curve and the resulting SPL response of a bass reflex enclosure. ----- below the first peak $f < f_1 < f_2$ $1 \times [1.0, 1.2] + 0.25 \times [1.0, -0.8] = [1.25, 1.0]$ The port mass is moving into the cabinet almost as much as the driver mass is moving into the room. The sound output almost cancels hence the 24 dB/octave roll off. ----- at the first peak $f = f_1$ $1 \times [1.0, 1.2] + 0 \times [1.0, -0.8] = [1.0, 1.2]$ ----- between the two peaks $f_1 < f < f_2$ $1 \times [1.0, 1.2] + 1 \times [1.0, -0.8] = [0.0, -2.0]$ The driver is not moving, $Z = R_e$. All of the sound is coming from the port! This is the minimum that can occur in the impedance curve. Since $f > f_1$ the phase reversal of the first mode occurs as described in the single degree of freedom paragraph above. ----- at the second peak $f = f_2$ $0.25 \times [1.0, 1.2] + 1 \times [1.0, -0.8] = [1.0, -0.8]$ Sound is coming from the driver and the port and is combining to produce the total SPL. ----- above the second peak $f > f_2$ $0.25 \times [1.0, 1.2] + -0.75 \times [1.0, -0.8] = [-1.0, 0.3]$ Both modes have a phase reversal in the displacement since $f > f_1$ and $f > f_2$. Again

sound is coming from the driver and the port and combining in the room, the driver output dominates the system SPL. Keeping this in mind, why not look at a typical impedance curve and a SPL curve (hopefully showing the driver, port, and summed SPL response) and compare what is shown and discussed above to see if it matches what is shown in at the various regions in the plotted data. Maybe the LDC has a set of curves, I don't remember. So what does this have to do with the transient response? Any transient signal can be transformed into the frequency domain using the Fourier Transform. The frequency content can now be displayed. Lets say that the transient signal has a large frequency content at frequencies around f_b and smaller inputs at other frequencies between 0 and 200 Hz. Remember that the input is to the driver via the voice coil and no force is exerted directly on the enclosure but must come from driver's cone motion. The smaller inputs at frequencies other than f_b will cause the driver to move small amounts. Driver motion definitely results. The big input at f_b will also cause the driver to move, the movement will be a combination of the two modes as shown above in the section labeled "between the two peaks". The force will split providing excitation to both modes. Both modes will be excited but they are out of phase and when combined the driver motions will tend to cancel resulting in a minimal displacement. There will be some motion present at f_b depending on the coupling of the two modes, but it will be smaller than one would expect based on the magnitude of the force being applied. The large damping associated with the driver will quickly (hopefully!) attenuate the motions so that a booming ringing response is avoided. OK, I have tried to relate the shape of the impedance curve to the physical motions of the driver and the air in the port to help explain what the system is doing and why the impedance curve has two peaks. I have made up a special case to try and help visualize the mechanics and physics of the problem. Then after all of that long babble, I have tried to present what a transient forcing function will excite. This is a huge over simplification of the problem but I think that it helps visualize what is physically going on. Please do not read this too closely and focus on technical nits to pick at, step back and try and visualize the systems motions and talk your way through the physical responses of the driver mass and the port mass. Everything is tied together since motions of the driver and port air masses generate the electrical impedance curve and the sound waves we hear. I am open to any questions, discussion or criticisms that might improve this explanation and my understanding of the impedance curve." I hope that helps, Martin
