Forum Acusticum 2011 - Aalborg - Denmark

Comparisons between models and measurements of the input impedance of brass instruments bells

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Abstract
This work is part of a project aiming at helping craftsmen to design and characterize their musical instruments. Starting from a given wind instrument shape, our objective consists in choosing the most relevant physical model able to predict the acoustical input impedance of this musical instrument once constructed. The modeling of bells in brass instruments is still problematic as the limits of plane wave approximation are known but no method is proven to give accurate results. The aim of the present paper is to compare the results given by different methods with measurements. Four different bells with known geometries are used in this study. These bells input impedances are calculated with a Boundary Element Method and with Transmission-Matrix Methods loaded with various radiation impedance models and based on axial or curvilinear abscissa. Surprisingly, a simple 1D wave propagation approximation based on curvilinear abscissa coupled with the radiation impedance of a pulsating portion of sphere gives results very close to the measurement.

PACS no. 43.75.Fg, 43.20.Mv

1. Introduction

This work is part of a project aiming at helping craftsmen to design and characterize their musical instruments. Starting from a given wind instrument shape, our objective consists in choosing the most relevant physical model able to predict the acoustical input impedance of this musical instrument once constructed. Musical acoustics has been investigated for a long time and a lot of models have been provided [1]. Nevertheless, the modeling of bells in brass instruments is still problematic as the limits of plane wave approximation are known but no method is proven to give accurate results.

Two effects have to be taken into account so as to give a complete model of the horn: the wave propagation and the radiation.

One of the simplest and most efficient method to calculate the propagation inside a waveguide is the Transmission-Matrix Method (TMM). This method approximates the instrument structure as a sequence of concatenated segments, cylinders or cones, each being mathematically represented as a 4x4 matrix in which the terms are complex-valued and frequency-dependent. With an entire instrument described as a transmission line, it is easy to calculate quantities at the input end, defined as the usual point of excitation, given quantities at the output end. The Transmission-Matrix Method which is used in this paper is described in the article of Caussé et al. [2].

This method proved to give results closed to the measurement for wind instruments of cylindrical geometry [3]. However, for horns, it is not possible to consider plane waves any more [4]. This is why Nederveen and Dalmont [5] propose a low frequency correction for the TMM in form of an additionnal impedance that takes into account the transverse flow inside the horn. Nevertheless, a problematic issue is that the wavefront is still unknown.

With regard to radiation, the problem of the acoustic radiation impedance of a cylindrical pipe is now well known. Cases of unflanged and infinitely flanged cylinder have been solved [6, 7]. These results were extended by Silva et al. [8] above the cutoff frequency of the first higher order mode. They gave a non-causal expression obtained by analytical and numerical fitting to reference results from Levine and Schwinger [6] for the unflanged case and extracted from the radiation impedance matrix given by Zorumski [9] for the infinite flanged case.

However, these radiation impedances cannot be used in every cases. Indeed, the shape of a horn has a strong influence on the acoustic wave propagation and radiation, to such an extent that it is no longer possible to use a plane radiation impedance. This is why Caussé et al. [2] have introduced a correction for the case of spherical waves, normalizing the Levine's impedance by the ratio A_p/A_s where A_p and A_s are respectively the planar and spherical areas. However, this constant ratio does not account for effects due to the curvature of the wavefront, such as modifications of resonance features or phase. In order to obtain a more adequate geometric model Hélie and Rodet [10] gave an analytical formulae for the radiation of a pulsating portion of a sphere.

2. Experiment set-up and bells measured

2.1. Experiment set-up

For the input impedance measurement, a set-up developed jointly by CTTM (Centre de Transfert de Technologie du Mans, France) and LAUM (Laboratoire d'Acoustique de l'Universite du Maine) is used [11]. In this apparatus, a small closed cavity in which a microphone measures a pressure p_1 (which gives an estimation of the volume velocity of the source), is connected to the back of a piezoelectric buzzer. The measured pipe is connected to the front of the buzzer via a small open cavity in which a second microphone measures a pressure p_2. Then the input impedance can be calculated from the transfer function between the two microphones.

The signal used as source is a logarithmic chirp signal of five seconds length (generated by the PC audio sound card) leading to a frequency resolution of 0.2 Hz, from 10 to 2500 Hz (until 4000 Hz for the long cone). Finally, the measurement is obtained by averaging three acquisitions. The entire apparatus is placed in an anechoic chamber whose temperature is estimated before and after the acquisitions by measuring the input impedance of a closed cylinder of length 624 mm and radius 10.9 mm. According to Macaluso and Dalmont in [12], the measurement set-up allows the determination of the resonance frequencies with an uncertainty of about 0.2%. Moreover, this measurement apparatus was first tested by the authors with simple known cases, in particularly with the cylinder mentionned before. The difference between the measurement and the model (TMM with axial abscissa and radiation with a finite flange from [13]) in frequency was less than 0.15% for all resonance peaks. This uncertainty allows to make a meaningful comparison between the different models and the horns measurements.

2.2. Bells measured

Four horns are studied in this paper. The first one is a tenor trombone bell which starts, once the water key and the slide removed, with a cylindrical section of 10.4-mm radius. It begins to flare modestly, terminating, in an abrupt flare to a radius of 110 mm, after a 568-mm length. The second one is a 609.6-mm trumpet horn which begins with a long part of cylinder of a 5.8-mm radius and ends at a radius of 61.1 mm. The last two bells are cones, one of whom is extended by a cylinder. The long cone is in fact the body of a soprano saxophone. The geometry was measured with an accuracy to within a hundreth of a millimeter with precision tools, either on the mandrel which was used to construct the trumpet bell, or directly on the horn itself for the others. These geometries are shown in Figure 1.

Their input impedance was measured six times, by removing the bell each time from the impedance sensor in order to study the reproductibility. The reproductibility error is about 0.2% which is of the same order than the measurement apparatus uncertainty. Consequently, measurements of these horns can be considered as a reference for the comparison with models.

3. Models

For crafstmanship applications it is important to have a reliable resolution that works for all geometries. That is why the method that seems to be appropriate for that case is a Transmission-Matrix Method applied to a bore approximated by only truncated cones or cylinders. Two propagation models, a plane and a spherical, are made from the TMM in order to study the particular case of horns acoustics where the wavefront is quasi-spherical. The plane model is calculated along the horn axis whereas the spherical one is computed along the bore profile (curvilinear abscissa). Two models of radiation are chosen to be loaded to the propagation models at the end of the horns: the plane non-causal unflanged model of Silva et al. [8] and the spherical second order high-pass model of Hélie and Rodet [10]. Two complete models are thus created: a plane model, made from the combination of the plane propagation and the plane radiation cited above, which is currently used in existing crafstmanship softwares, and a spherical model (made from the combination of both remaining models).

A benchmark test was launched some months ago to compare other methods with these two models and is still in progress. Two methods have been tested for the trombone horn for the moment. A Boundary Element Method (BEM) is computed with two different meshes, one mesh directly realised on the 3D geometry and another by meshing the bore (showed on Figure 1) and considering an axisymmetric problem. Then it

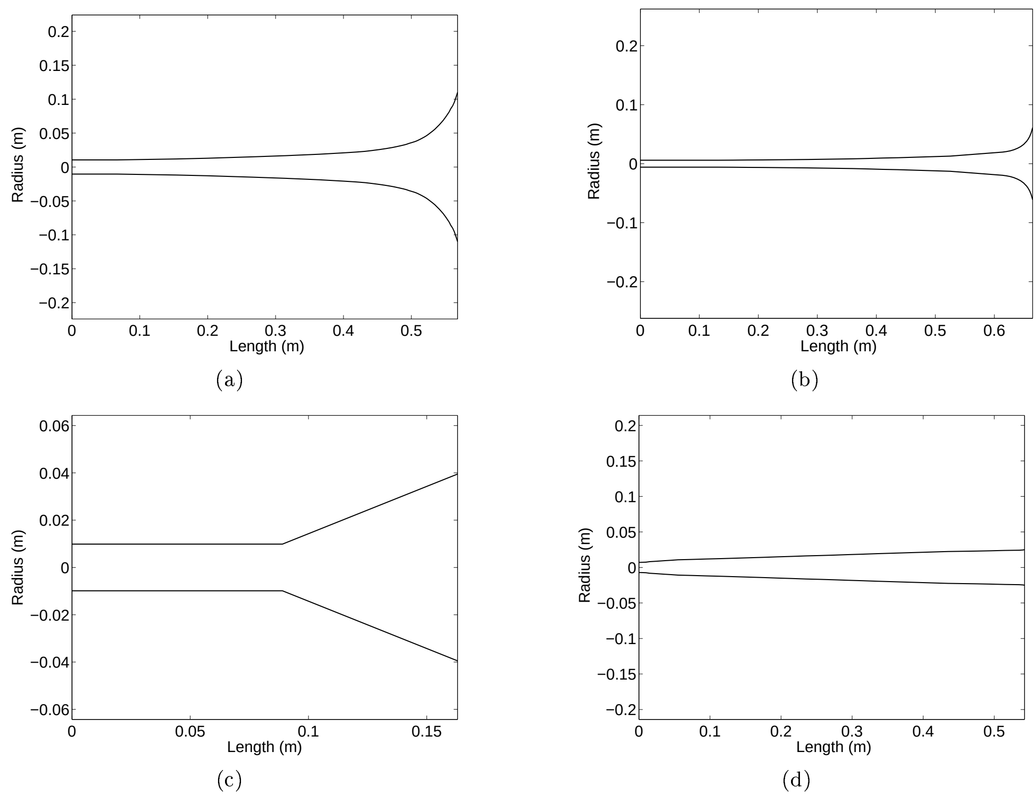

Figure 1. Bores of the horns studied: (a) trombone horn, (b) trumpet horn, (c) cone with cylinder and (d) long cone.

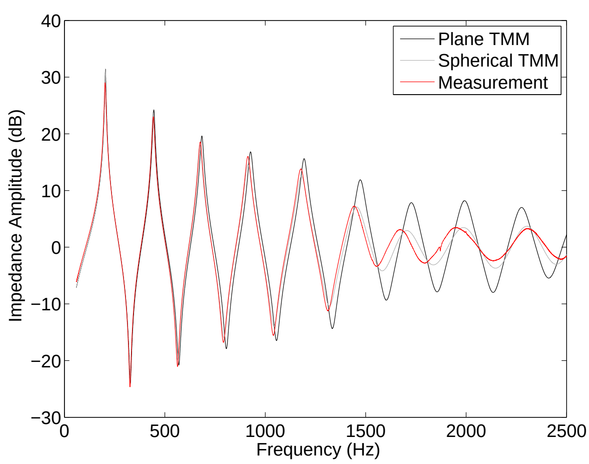

Figure 2. Comparison between the measurement of the trumpet bell input impedance (in red) and the two TMM models: the plane (in black) and the spherical (in grey).

is computed with Sysnoise [15]. The other method is a Multimodal Method explained by Amir et al. [16].

4. TMM models for the trumpet

The behaviour of the input impedance of the trumpet horn in Figure 2 is different below and above the cut-off frequency F_c of the bell which is around 1300 Hz (see Benade [14]). Indeed, below that frequency, resonance peaks are sharp since almost all acoustic waves are reflected at the end of the bell, whereas after, the radiation is more important and peaks decrease significantly. It is clear that at high frequency the spherical model is the closest to the measurement since it realises a better impedance adaptation than the planar model. However, below the cutoff frequency, models need to be accurately compared with regard to impedance resonance peaks. These peaks are defined by three criteria: the frequency, the amplitude and the quality factor. In order to help crafstmen to design their musical instruments, the frequency criterion is the more important, as it is linked to the instrument tuning. The amplitude and the quality factor have a less audible effect as they influence the instrument timbre. Therefore, this comparison is only done on the frequency and the amplitude of all peaks which are precisely determined with a peak fitting technique using a least square method.

Results in Table I show that the spherical model gives resonance frequencies closer to the measurement than the planar one. That supports the hypothesis of the quasi-sphericity of wavefronts which was experimentally established in low frequency range by Jansson and Benade [4]. The measured input impedance is expected to be located between the plane and the spherical model. It is actually the case for the sec-

Table I. Differences between TMM models and measurement at low frequency for the trumpet horn

Resonance	Measurement	Plane	Spherical
1st	Fr=202.87 Hz	0.83%	0.56%
	A=29.00 dB	2.42 dB	2.40 dB
2nd	Fr=443.09 Hz	0.52%	-0.37%
	A=23.03 dB	1.20 dB	0.77 dB
3rd	Fr=676.53 Hz	1.18%	-0.27%
	A=18.59 dB	1.04 dB	-0.49 dB
4th	Fr=913.41 Hz	1.48%	0.07%
	A=16.00 dB	0.84 dB	-1.38 dB
5th	Fr=1177.20 Hz	1.38%	0.26%
	A=13.83 dB	1.80 dB	-1.27 dB

Table II. Differences between TMM models and measurement at low frequency for the trombone horn

Resonance	Measurement	Plane	Spherical
1st	Fr=241.43 Hz	2.7%	1.8%
	A=35.05 dB	0.41 dB	-0.02 dB
2nd	Fr=517.22 Hz	2.8%	0.6%
	A=23.89 dB	1.53 dB	-2.52 dB

ond and third impedance peaks, with a measurement closer to the spherical model. The low frequency of the last resonances (lower than predicted by both plane and spherical models) can be explained by the influence of the radiation that starts to be more important around these frequencies. No explanation can be given about the reasons why the first measured resonance has a frequency lower than predicted by both models. Moreover it was checked (see section 2.1) that it was not related to the measurement apparatus.

5. Benchmark test for the trombone

The comparison between the TMM models and the measurement is now applied to the trombone horn. Table II shows that the spherical TMM model is the closer to measurement than the plane one. Furthermore, as it was established in section 4, the spherical model realises such an impedance adaptation that it approximates better the measurement. Consequently, there is no point in comparing the plane model to the others.

Owing to the long computation time for the BEM and Multimodal method, resolution is rough and peaks are not well defined, consequently it is not possible to compare the models above the cutoff frequency for these methods (see Figure 3). For the moment we

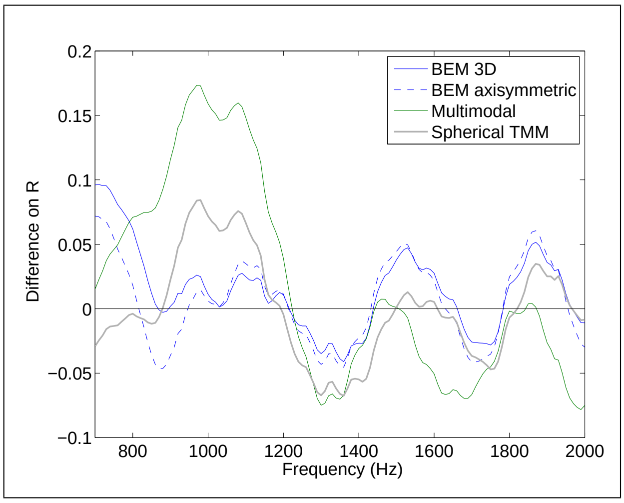

Figure 4. Difference between the modulus of the reflection coefficient R calculated with each of the presented methods and the modulus of the reflection coefficient from the measurement.

can say that the measurement still gives a first resonance frequency above all the models and that the difference between models and measurement is less than 2% for the second peak. It is more interesting here to analyse high frequencies by looking at the reflection coefficient.

Figure 4 confirms that the multimodal method is not optimum for the moment. Indeed, the horn is supposed to come out onto an infinite flange which does not represent ideally the reality. A Perfectly Matched Layer method (PML) applied to an unflanged case is considered to compensate for that problem. In a BEM, no approximation is done on the radiation, that is why the 3D model solved with a BEM is the closest to the measurement (differences between the 3D and the axisymmetric models are only due to meshes). This comparison with other methods is still in progress, but the first results show that, even if it is possible to have a better accuracy, results from the spherical TMM are of the same order of magnitude as other methods.

6. Results for the two cones

The small cone extended with a cylinder is used as textbook case since the peak located toward the trough of the impedance curve around 1500 Hz in Figure 5 shows the interferences that appear with this kind of geometry. Indeed, at the discontinuity between the cylinder and the cone, some waves are reflected whereas others keep on propagating through the cone. In the cone, as for the two other horns, the first measured resonance frequency is lower than these calculated with both TMM models. The spherical TMM model is closer to measurement in frequency, especially at the impedance troughs. On the contrary, the long cone gives results (see Figure 6) which are different from the previous ones.

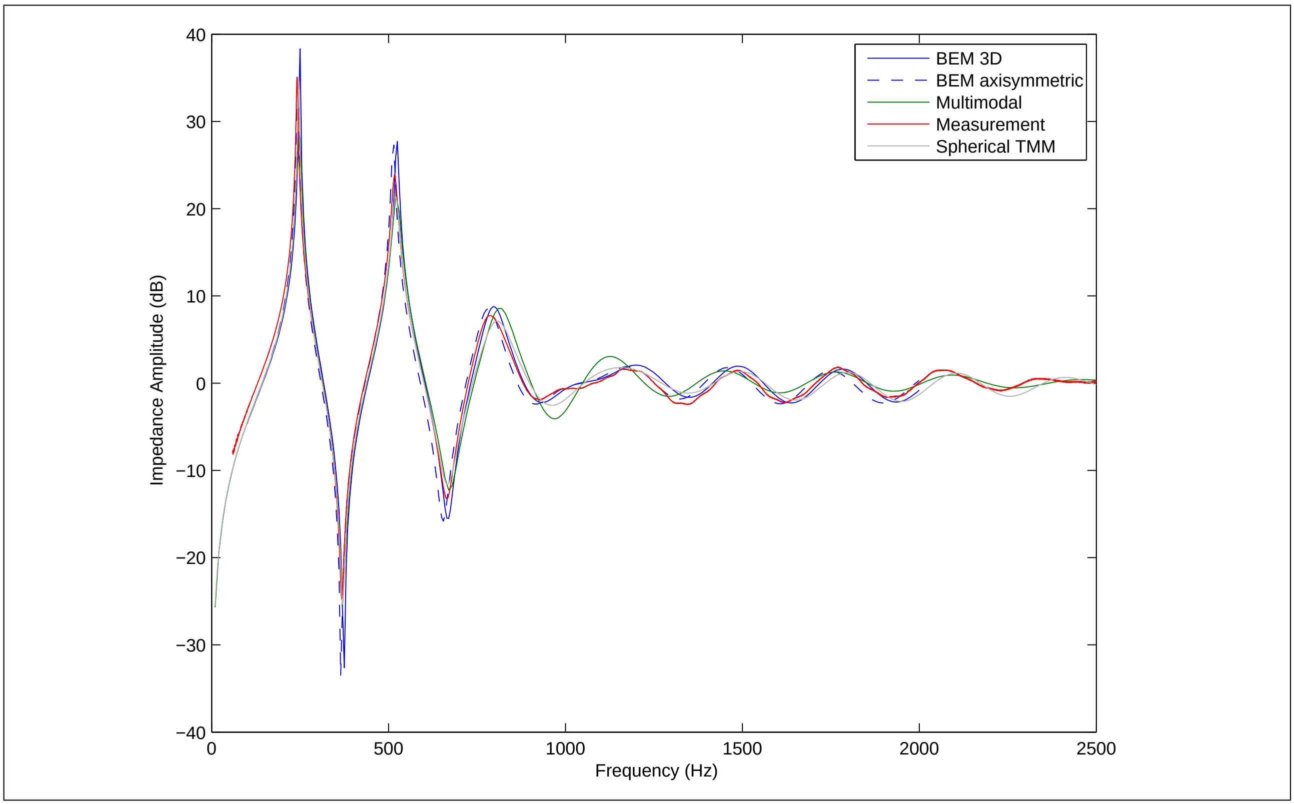

Figure 3. Comparison between the measurement of the input impedance of the trombone horn and results from different resolution methods.

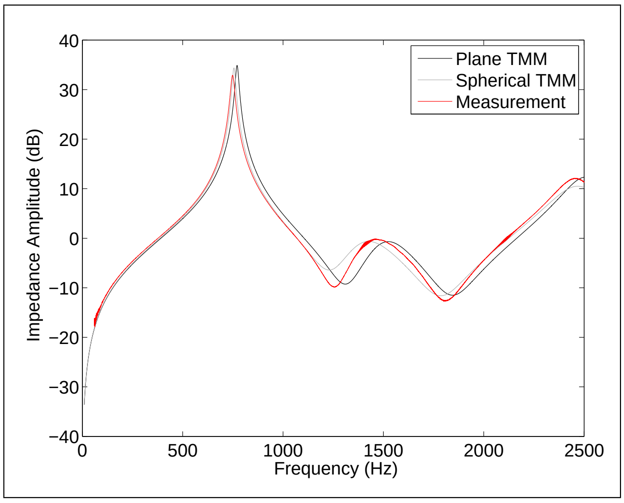

Figure 5. Comparison between the measurement of the short cone input impedance (in red) and the two TMM models: the plane (in black) and the spherical (in grey).

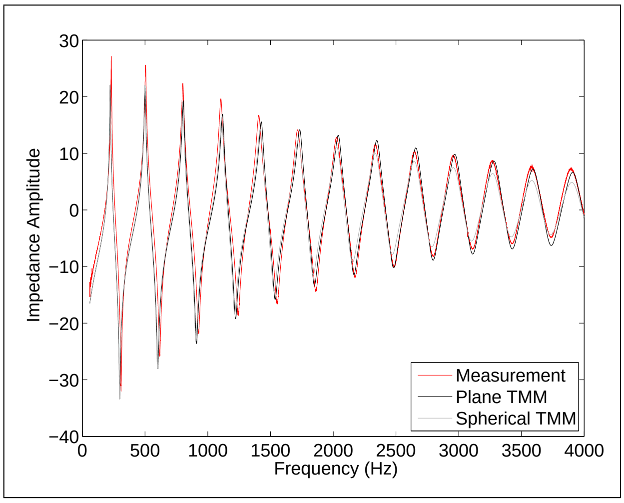

Figure 6. Comparison between the measurement of the long cone input impedance (in red) and the two TMM models: the plane (in black) and the spherical (in grey).

Contrary to the other horns, and as it can be seen on Table III, the first measured resonance frequency is higher than that computed with both models. This difference may be an error due to a wrong discontinuity correction from the measurement apparatus normally provided for a cylinder. Nevertheless, even if the difference between the measurement and the models decreases for the first peak by adding a 6 cm-long-cylinder at the cone input, the first measured resonance is still higher than the ones computed. Moreover, for the first peak, the difference on the frequency between models and measurement is more than 4% whereas for other horns it does not rise upon 2%. A 5 dB difference between measurement and models for that first peak is also striking. There is actually a larger difference in amplitude for the first resonance of the trumpet horn too but it is only 2.4 dB.

Table III. Differences on the resonance frequencies between TMM models and measurement for the long cone

Resonance	Measurement	Plane	Spherical
1st	Fr=230.80 Hz	-3.73%	-4.18%
2nd	Fr=504.08 Hz	-0.85%	-1.37%
3rd	Fr=801.58 Hz	0.37%	-0.17%
4th	Fr=1104.50 Hz	1.13%	0.53%
5th	Fr=1406.60 Hz	1.40%	0.77%
6th	Fr=1715.85 Hz	0.97%	0.32%
7th	Fr=2026.98 Hz	0.61%	0%
8th	Fr=2335.69 Hz	0.46%	-0.08%
9th	Fr=2647.14 Hz	0.39%	-0.09%
10th	Fr=2956.23 Hz	0.41%	0.01%
11th	Fr=3267.07 Hz	0.40%	0.07%

Except for the first two resonances that pose a problem, the spherical TMM model gives frequencies closer to the measurement. Nevertheless, at high frequency the plane TMM approximates better the amplitudes. Indeed, it seems that the impedance break at the end of the cone is more important in reality that what is supposed by the spherical model of radiation. Furthermore, the radiation of a pulsating portion of sphere supposes the presence of a spherical flange which could influence the results, but this is not discussed here.

Finally, even if a cone seems to be the simplest case to study, it is the one that raises the most questions.

7. Conclusion

This article shows that a simple and fast method as the TMM can lead to quite accurate results for the trumpet and trombone horns studied. As well as measurements, this method is also compared to two other computation methods. That confirms the validity of the TMM which gives results of the same order of magnitude than these other methods. It is planned to carry on with the horn benchmark test and to extend it to methods of Finite Element (FEM) and Finite-Difference Time-Domain (FDTD). Results obtained with the TMM are promising for musical instruments crafstmanship as this method does not require a strong computing power (any craftsman should be able to use the designed software with his own personal computer) and calculations are fast. It is a significant alternative to the plane wave approximation currently used.

In spite of these convincing results, some questions still remain, especially when looking at the measurement of a cone.

Acknowledgement

This research was funded by the French National Research Agency ANR within the PAFI project (Plateforme d'Aide à la Facture Instrumentale in French). The authors would like to thank all participants for all discussions and interesting work done together. Thank you to P. Herzog and T. Scotti from LMA-CNRS for their BEM results and to S. Félix from LAUM-CNRS for the Multimodal Method. The authors also want to thank T. Hélie from IRCAM-CNRS for interesting discussions about this work.

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